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Higgs Sector of the Minimal Left-Right Symmetric Model

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We perform an exhaustive analysis of the most general Higgs sector of the minimal left-right symmetric model (MLRM). We find that the CP properties of the vacuum state are connected to the Higgs spectrum: if CP is broken spontaneously, the MLRM does not approach the Standard Model in the limit of a decoupling left-right symmetry breaking scale. Depending on the size of the CP phases scenarios with extra non-decoupling flavor-violating doublet Higgses or very light SU(2) triplet Higgses emerge, both of which are ruled out by phenomenology. For zero CP phases the non-standard Higgses decouple only if a very unnatural fine-tuning condition is fulfilled. We also discuss generalizations to a non-minimal Higgs sector.

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Introduction. Left-right symmetric models are extensions of the Standard Model (SM) based on the gauge group $\mathrm{SU(2)_R\times SU(2)_L\times U(1)_{B-L}}$ [1]. The right-handed fermion fields are $\mathrm{SU(2)_R}$ doublets and parity P is an exact symmetry of the Lagrangian. At a high scale v_R well above the electroweak breaking scale $\mathrm{SU(2)_R\times SU(2)_L\times U(1)_{B-L}\times P}$ is spontaneously broken to the SM gauge group $\mathrm{SU(2)_L\times U(1)_Y}$. The U(1) charges have a physical interpretation as the difference B−L of baryon and lepton number. The hypercharge Y, which is an ad-hoc quantum number of the SM, emerges as the combination $\mathrm{Y= T_{3,R} + (B-L)/2}$, where $\mathrm{T_{3,R}}$ is the third component of the right-handed isospin. The Higgs sector of the MLRM consists of two Higgs triplets Δ_R, Δ_L and a bidoublet Φ. The neutral component of Δ_R acquires a vacuum expectation value (VEV) v_R, which breaks the $\mathrm{SU(2)_R}$ and P symmetries. The bidoublet Φ breaks the electroweak symmetry down to $U(1)_{em}$. The choice of Higgs triplets $\Delta_{L,R}$ rather than doublets permits a Majorana mass term and thereby small neutrino masses via the see-saw mechanism [2]. This requires v_R to be very high, typically of order $10^{10}-10^{15}$ GeV.

Spontaneous CP violation. Models with explicit CP violation suffer from a general problem: a CP-non-invariant Lagrangian usually contains too many sources of CP violation. In most extensions of the SM, especially in the MSSM, this problem becomes very severe: some CP-violating phases must be fine-tuned to comply with the observed smallness of CP violating observables. Models with spontaneous CP violation (SCPV) are therefore an attractive alternative, because their only few sources of CP violation are the complex phases of the VEVs of Higgs fields. The MLRM with SCPV has recently attracted new attention in the context of CP phenomenology probed in current experiments [3,4]. The MLRM with SCPV has the attractive feature that C,P and T are all exact symmetries of the Lagrangian. Both the MLRM with and without SCPV have been studied extensively. Yet its complicated Higgs sector has never been analyzed completely. In [3–5] the case of sizable CP-violating phases was considered aiming at the explanation of the observed CKM CP phase through SCPV. In [6,7], however, large CP-violating phases of the VEVs have been discarded because of fine-tuning arguments. We will clarify this point in the following.

Model. The $\mathrm{SU(2)_L\times SU(2)_R\times U(1)_{B-L}}$ charge assignments for the quark and lepton multiplets are $Q_L(1/2,0,1/3)$, $Q_R(0,1/2,1/3)$, $L_L(1/2,0,-1)$ and $L_R(0,1/2,-1)$. The Higgs multiplets are

$$\Phi=\begin{pmatrix}\phi_1^0 & \phi_1^+\\ \phi_2^- & \phi_2^0\end{pmatrix},\quad \Delta_{L,R}=\begin{pmatrix}\delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R}\\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2}\end{pmatrix}.$$

They transform under $\mathrm{U}_{L,R}\in\mathrm{SU(2)_{L,R}}$ as $\Phi\to U_L\Phi U_R^\dagger$, $\Delta_L\to U_L\Delta_L U_L^\dagger$ and $\Delta_R\to U_R\Delta_R U_R^\dagger$. The Higgs fields transform under parity as $\Delta_L\leftrightarrow\Delta_R$ and $\phi\leftrightarrow\phi^\dagger$. Their charge conjugation transformation reads:

$$\mathrm{C}_\phi:\quad \phi\leftrightarrow\tilde\phi=\tau_2\phi^*\tau_2,\quad \mathrm{C}_\Delta:\quad \Delta\leftrightarrow\tilde\Delta=\tau_2\Delta^*\tau_2. \quad (1)$$

The most general C×P-invariant Higgs potential is [6]

$$\begin{aligned}
V(\Delta_R,\Delta_L,\Phi) \;=\; & -\mu_1^2\mathrm{Tr}(\phi^\dagger\phi)-\mu_2^2\left[\mathrm{Tr}(\tilde\phi\phi^\dagger)+\mathrm{Tr}(\tilde\phi^\dagger\phi)\right]\\
&-\mu_3^2\left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)+\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right]+\lambda_1\left[\mathrm{Tr}(\phi\phi^\dagger)\right]^2\\
&+\lambda_2\left\{\left[\mathrm{Tr}(\tilde\phi\phi^\dagger)\right]^2+\left[\mathrm{Tr}(\tilde\phi^\dagger\phi)\right]^2\right\}+\lambda_3\left[\mathrm{Tr}(\tilde\phi\phi^\dagger)\mathrm{Tr}(\tilde\phi^\dagger\phi)\right]\\
&+\lambda_4\left\{\mathrm{Tr}(\phi\phi^\dagger)\left[\mathrm{Tr}(\tilde\phi\phi^\dagger)+\mathrm{Tr}(\tilde\phi^\dagger\phi)\right]\right\}\\
&+\rho_1\left\{\left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\right]^2+\left[\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right]^2\right\}\\
&+\rho_2\left[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)+\mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)\right]\\
&+\rho_3\left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right]\\
&+\rho_4\left[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)+\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R)\right]\\
&+\alpha_1\left\{\mathrm{Tr}(\phi\phi^\dagger)\left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)+\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right]\right\}\\
&+\alpha_2\left[\mathrm{Tr}(\phi\tilde\phi^\dagger)+\mathrm{Tr}(\phi^\dagger\tilde\phi)\right]\left[\mathrm{Tr}(\Delta_R\Delta_R^\dagger)+\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\right]\\
&+\alpha_3\left[\mathrm{Tr}(\phi\phi^\dagger\Delta_L\Delta_L^\dagger)+\mathrm{Tr}(\phi^\dagger\phi\Delta_R\Delta_R^\dagger)\right]
\end{aligned}$$

$$+\beta_1\left[\mathrm{Tr}(\phi\Delta_R\phi^\dagger\Delta_L^\dagger)+\mathrm{Tr}(\phi^\dagger\Delta_L\phi\Delta_R^\dagger)\right]$$
$$+\beta_2\left[\mathrm{Tr}(\tilde\phi\Delta_R\phi^\dagger\Delta_L^\dagger)+\mathrm{Tr}(\tilde\phi^\dagger\Delta_L\phi\Delta_R^\dagger)\right]$$
$$+\beta_3\left[\mathrm{Tr}(\phi\Delta_R\tilde\phi^\dagger\Delta_L^\dagger)+\mathrm{Tr}(\phi^\dagger\Delta_L\tilde\phi\Delta_R^\dagger)\right]. \qquad (2)$$

Here all coefficients are real. We discuss the case of a CP non-invariant Higgs potential later. The coefficients in (2) must be such that V has a nontrivial minimum which leaves $U(1)_{em}$ unbroken. By using the broken symmetries one can arrange the VEVs such as [1,6]

$$\langle\phi_1^0\rangle=\frac{k_1}{\sqrt2},\ \langle\phi_2^0\rangle=\frac{k_2}{\sqrt2}e^{i\alpha},\ \langle\delta_L^0\rangle=\frac{v_L}{\sqrt2}e^{-i\theta},\ \langle\delta_R^0\rangle=\frac{v_R}{\sqrt2},$$

with real and positive $v_{L,R}$ and $k_{1,2}$. If β_i,$\rho_i=\mathcal{O}(1)$, the condition $v_R\gg k_{1,2}$ automatically enforces $k_1k_2/(v_Lv_R)$ to be of order 1. This VEV see-saw mechanism [6] suppresses v_L, as needed to comply with the data on the electroweak ρ parameter. The Yukawa Lagrangian reads

$$-L_Y=\bar Q_L\,\hat F\,\phi\,Q_R+\bar Q_L\,\hat G\,\tilde\phi\,Q_R+h.c.. \qquad (3)$$

Here $\hat F$ and $\hat G$ are 3×3 matrices in flavor space. If L_Y conserves CP, one can choose them real and symmetric. Since ϕ_1^0 and ϕ_2^0 couple to both up and down quarks there will be flavor-changing neutral couplings. Here it is useful to define [5]

$$\begin{pmatrix}\phi_-^0\\ \phi_+^0\end{pmatrix}=\begin{pmatrix}\cos\beta & \sin\beta\,e^{i\alpha}\\ -\sin\beta\,e^{-i\alpha} & \cos\beta\end{pmatrix}\begin{pmatrix}\phi_1^0\\ \phi_2^{0*}\end{pmatrix}, \qquad (4)$$

with $\tan\beta=k_2/k_1$. Analogously we define (ϕ_-^+,ϕ_+^+). The imaginary part of the flavor-conserving field ϕ_-^0 becomes a component of the Goldstone bosons eaten by Z's. The orthogonal combination ϕ_+^0 has flavor-changing neutral couplings. For example the flavor-changing couplings to the down quarks are given by

$$\mathcal{L}^{FC\phi^0d}=\sqrt2\frac{k_+}{k_-^2}\phi_+^{0*}\bar D_LV_L^\dagger M_uV_RD_R, \qquad (5)$$

where $k_\pm^2=k_1^2\pm k_2^2$ with the electroweak breaking scale $k_+\simeq246\,\mathrm{GeV}$. P and CP invariance imply $|V_L|=|V_R|$ and calculable phases for the left and right-handed CKM matrices. If P and CP are broken spontaneously, the masses of the flavor-violating Higgses must exceed 10 TeV from phenomenology [4]. If one relaxes the CP invariance of L_Y, the phases of V_R become independent of V_L and generically this lower bound becomes much stronger. Hence the mass of the flavor-changing Higgs must be determined by v_R rather than k_+. The CP-violating complex phase α enters the quark mass matrix through the Yukawa interactions in (3). If L_Y in (3) is chosen to conserve CP, the CKM CP violation stems solely from $\alpha\neq0$. This case requires that $k_1k_2(\sin\alpha)/k_-^2\approx m_b/m_t$ [3–5]. In [3–5] this has been achieved by choosing $k_2/k_1\le\mathcal{O}(m_b/m_t)$ and $\alpha=\mathcal{O}(1)$.

We next decompose the Higgs fields into real and imaginary parts: $\phi_1^0=(\phi_1^{0r}+i\phi_1^{0i}+k_1)/\sqrt2$, $\phi_2^0=(\phi_2^{0r}+i\phi_2^{0i}+k_2)\exp(i\alpha)/\sqrt2$ and analogously Δ_L^0 and Δ_R^0. V in (2) is minimized by solving the equations

$$\frac{\partial V}{\partial\phi_1^{0r}}=\frac{\partial V}{\partial\phi_2^{0r}}=\frac{\partial V}{\partial\phi_2^{0i}}=\frac{\partial V}{\partial\Delta_R^{0r}}=\frac{\partial V}{\partial\Delta_L^{0r}}=\frac{\partial V}{\partial\Delta_L^{0i}}=0. \quad (6)$$

In general (6) expresses six chosen parameters of V in terms of the remaining parameters and k_1,k_2,α,v_R,v_L and θ. Yet there is an important exception: if the parameters in V are such that CP remains unbroken, the complex phases α and θ are zero and V is quadratic in $\phi_{1,2}^{0i}$. The rank of (6) then collapses to 4 allowing us to solve for only 4 parameters in terms of k_1,k_2,v_R and v_L. For generic choices of the parameters in V the polynomial equations in (6) will not generate the desired gauge hierarchy $v_R\gg k_+$. This relation must be encoded in V: either some ratio of dimensionful parameters μ_i^2 or some coupling or a combination of couplings must be chosen small to define the ratio k_+^2/v_R^2. From this consideration it is clear that fine-tuning is unavoidable. Parameters fine-tuned to small values must be protected by an approximate symmetry, otherwise the corresponding solution becomes unstable under radiative corrections.

Our strategy is to expand the solutions of (6) and the Higgs mass matrices in terms of $\epsilon=\max\{k_+/v_R,v_L/k_+\}$. Thereby we determine the Higgs spectrum in the decoupling limit. We will see that there are different possibilities to generate the gauge hierarchy: depending on which parameter is chosen to be of order ϵ^2, different low energy models emerge in the decoupling limit. We first use the derivatives with respect to $\phi_{1,2}^{0r}$ and Δ_R^{0r} in (6) to find

$$\frac{\mu_1^2}{v_R^2}\simeq\frac{\alpha_1}{2}-\frac{\alpha_3k_2^2}{2k_-^2},\quad \frac{\mu_2^2}{v_R^2}\simeq\frac{\alpha_2}{2}+\frac{\alpha_3k_1k_2}{4k_-^2\cos\alpha},\quad \frac{\mu_3^2}{v_R^2}\simeq\rho_1. \quad (7)$$

These relations are valid in all scenarios discussed in the following. Here and in the following "$\simeq$" means "equal up to corrections of order ϵ^2." Next we calculate the mass matrices from the second derivatives of V with respect to the Higgs fields and insert the results for the six parameters found from (7). This step gives us a 2×2 mass matrix for the doubly charged Higgs fields, a 4×4 matrix for the singly charged Higgses and an 8×8 matrix for the neutral ones. The latter two contain 2 zero eigenvalues each corresponding to the pseudo-Goldstone modes eaten by the left- and right-handed vector bosons. A pivotal role for the mass spectrum is played by the term involving α_3 in V, it is the only term which generates a mass splitting of order v_R^2 between the bidoublet fields:

$$\alpha_3\mathrm{Tr}(\phi^\dagger\phi\Delta_R\Delta_R^\dagger)\to\alpha_3|v_R|^2(|\phi_2^+|^2+|\phi_2^0|^2)/2. \qquad (8)$$

Here the components of $\phi_\pm=(\phi_\pm^0,\phi_\pm^+)$ are defined in (4). Up to corrections of order ϵ^2 the fields of ϕ_- become components of the Goldstones eaten by W_L and Z_L and of light Higgs particles, whose masses are of order k_+ or smaller. Hence if $\alpha_3=\mathcal{O}(1)$, the bidoublet components in ϕ_+, acquire masses of order v_R. However, we will be frequently lead to scenarios with $\alpha_3 = \mathcal{O}(\epsilon^2)$. Then all entries for the bidoublet mass matrices are at most of order k_+^2. From (2) one easily verifies that the terms which mix bidoublet and triplet fields are $\mathcal{O}(v_R k_+)$ or smaller. This form of the mass matrices implies that the neutral (charged) Higgs sector has at least four (two) physical Higgs masses of order k_+. Up to terms of order ϵ the corresponding mass eigenstates are bidoublet fields. That is, for $\alpha_3 = \mathcal{O}(\epsilon^2)$ one encounters a two-Higgs doublet model (2HDM) in the decoupling limit $v_R \to \infty$. In view of the flavor-changing couplings in (5) such scenarios are unacceptable.

Scenarios with $v_L = 0$. We will study the $v_L = 0$ scenario here for two reasons: first, it has been used extensively in the literature [6,9,10], and second, most of the characteristic features of the general case can be studied from this simplified case. In addition to (7) the minimization conditions in (6) give:

$$\alpha_3 = 4\tilde{\lambda}\frac{k_-^2}{v_R^2}, \quad \beta_1 = -2\beta_3\frac{k_2}{k_1}\cos\alpha, \quad \beta_2 = \beta_3\frac{k_2^2}{k_1^2}, \qquad (9)$$

where $\tilde{\lambda} = 2\lambda_2 - \lambda_3$. It must be clear that the scenario with $v_L = 0$ is singular, because the six equations in (6) involve only 4 parameters k_1,k_2,α and v_R. After eliminating k_1,k_2,α and v_R from (9) one finds two relations between the coefficients of V. In [6,9,10] this problem has been circumvented by choosing $\beta_{1,2,3} = 0$. Since there is no suitable symmetry, this scenario cannot be stabilized after renormalization. In particular the desired Majorana couplings induce non-zero β_i-terms at the loop level. After eliminating the μ_i's with (7) we find the physical eigenvalues of the singly charged mass matrix as

$$M_1^{+2} \simeq \frac{\alpha_3}{2}v_R^2\frac{k_+^2}{k_-^2}, \quad M_2^{+2} \simeq \frac{v_R^2}{2}(\rho_3 - 2\rho_1). \qquad (10)$$

Next we use (9) to eliminate α_3. Since α_3 is of order ϵ^2, this immediately implies that M_1^+ in (10) is of order k_+, not of order v_R. (The smallness of α_3 can be motivated by an approximate discrete symmetry: V is invariant under $C \equiv C_\phi \circ C_\Delta$ (defined in (1)). Demanding invariance of V under $C_\phi \times C_\Delta$ implies $\alpha_3 = 0$ (and $\beta_2 = \beta_3$).) The smallness of M_1^+ simply reflects our finding that $\alpha_3 = \mathcal{O}(\epsilon^2)$ leads to a 2HDM in the decoupling limit, as discussed after (8). The peculiar result for α_3 stems from $\partial V/\partial\phi_2^{0i}$ in (7). The mechanism here is the following: first the heavy scale v_R is defined by the size of the dimensionful parameters μ_i in V. Then the first equation in (9) tells us that in the chosen scenario (with $v_L = 0$ and $\alpha \neq 0$) the electroweak scale is defined by $k_+^2 = \mathcal{O}(\alpha_3 v_R^2)$. Choosing $\alpha_3 = \mathcal{O}(1)$ would fail to produce the desired gauge hierarchy $k_+ \ll v_R$. The important lesson is that the same small parameter α_3, which defines the ratio $k_+/v_R = \mathcal{O}(\epsilon)$ through (9), enters the physical Higgs masses in (10). Only after eliminating α_3 via the minimization conditions (9) the smallness of $M_1^+ = \mathcal{O}(k_+)$ becomes transparent. From the discussion preceding (8) we conclude that in the limit $v_R \to \infty$ the bidoublet does not decouple. What phenomenologically matters, is of course the masses of the FCNC Higgses. We have calculated the 8×8 neutral mass matrix and have indeed verified that there are FCNC Higgs masses of order $\alpha_3 v_R^2$ (or explicitly of order k_+^2). Even if $\tilde{\lambda}$ in (9) is stretched to the largest values compatible with perturbation theory, the FCNC Higgs masses are way too small to comply with the precision data from flavor physics. In addition to the unacceptable light FCNC Higgs a fine-tuning problem emerges in (7): the terms involving α_3 are now sub-leading in ϵ, the dependence on α is lost and the three parameters $\mu_{1,2,3}$ only depend on v_R, up to $\mathcal{O}(\epsilon^2)$ corrections. The last two equations in (7) now require that $\mu_3^2/\mu_i^2 \simeq 2\rho_1/\alpha_i$ for $i = 1, 2$.

A qualitatively new scenario, which has been studied extensively in [6,9,10], is obtained, if one chooses $\alpha = 0$. Now there is no SCPV and the rank of (6) collapses to four, because all equations are real. In particular the first equation in (9) is now absent and no restrictions on α_3 occur! $\alpha_3 = \mathcal{O}(1)$ is now possible, and for this choice we can arrange for a SM-like Higgs spectrum in the decoupling limit.

We demonstrate the impact of α on the mass spectrum for the case $\beta_i = 0$, which can be nicely seen from the mass term of $\phi_2^{0i} = \mathrm{Im}\,\phi_2^0$. After using (7) one finds

$$\begin{aligned}
&\phi_2^{0i}\left.\frac{\partial V}{\partial\phi_2^{0i}}\right|_{\phi_2^{0i}=0} + \frac{\phi_2^{0i\,2}}{2}\left.\frac{\partial^2 V}{\partial\phi_2^{0i\,2}}\right|_{\phi_2^{0i}=0} = \\
&\quad \phi_2^{0i}k_2\sin\alpha\left[\frac{\alpha_3}{2}v_R^2 - 2(2\lambda_2 - \lambda_3)k_-^2\right] \\
&\quad +\phi_2^{0i\,2}\left[\frac{\alpha_3}{4}v_R^2 + k_2^2\sin^2\alpha - k_-^2(2\lambda_2 - \lambda_3)\right]. \qquad (11)
\end{aligned}$$

For $\alpha \neq 0$ the fourth minimization equation (w.r.t. ϕ_2^{0i}) enforces the linear term to vanish yielding the condition for α_3 in (9). Then the low energy model is a 2HDM. If $\alpha = 0$, however, the linear term is zero automatically, and α_3 can be of order 1, so that both charged Higgs boson masses in (10) are naturally of order v_R^2. Put conversely, $\alpha_3 = \mathcal{O}(1)$ implies $\alpha = 0$ and a heavy second Higgs multiplet, while SCPV requires a small α_3 and thereby implies a second light doublet. Hence the CP properties of the vacuum state are connected to the Higgs spectrum in the decoupling limit. We further stress that the scenario with $\alpha = 0$ cannot be obtained by taking the limit $\alpha \to 0$ from the general case. For $\alpha = 0$ and $\alpha_3 = \mathcal{O}(1)$ we find a SM-like Higgs spectrum for $v_R \to \infty$ in agreement with [6,9,10]. However, we again face a fine-tuning problem, because in (7) the three equations only involve two parameters v_R and k_2/k_1. By eliminating v_R and k_2/k_1 from (7) one easily finds a relation between $\mu_{1,2,3}$, $\alpha_{1,2,3}$ and ρ_1 which cannot be justified by any symmetry.

General Scenarios. Next we will study general scenarios with $v_L \neq 0$. We solve the minimization conditions for μ_1^2, μ_2^2, ρ_1, ρ_3, β_1, and β_2. The solutions for μ_1, μ_2 and ρ_1 can be found in (7). For ρ_3, β_1 and β_2 we find:

$$\rho_3 = \frac{\alpha_3 \sin^2\alpha\, k_1^2 k_2^2}{v_L^2\left[k_1^2\sin^2\theta - k_2^2\left(\sin^2\alpha + \sin^2(\alpha+\theta)\right)\right]} + \mathcal{O}(\epsilon^0),$$
$$\beta_1 \simeq \rho_3 \frac{v_L v_R}{k_1 k_2}\frac{\sin\theta}{\sin\alpha}, \qquad \beta_2 \simeq -\beta_1 \frac{k_2}{k_1}\frac{\sin(\alpha+\theta)}{\sin\theta}. \quad (12)$$

If both α and $f \equiv k_1 k_2/(v_L v_R)$ are of order 1, one faces non-perturbatively large couplings β_1,β_2,$\rho_3 = \mathcal{O}(\epsilon^{-2})$, unless $\alpha_3 = \mathcal{O}(\epsilon^2)$. (12) can be viewed as a see-saw relation between $1/\alpha_3$ and β_1, ρ_3. The pivot is proportional to $\sin\alpha$, so that the effect vanishes in the absence of the CP-violating phase. Its origin can be traced back to the terms in V which involve ϕ_2^{0i}, as in (11). Hence we confirm the findings of [6,7] that $\alpha = \mathcal{O}(1)$ and $k_1 \sim k_2$ enforces the smallness of parameters in the potential. However, this feature also appears in all other scenarios, as we discussed above. Hence an exhaustive investigation of the Higgs sector requires to consider all these possibilities that CP phases, ratios of VEVs and some of the Higgs couplings scale with certain powers of ϵ. We have performed a complete analysis of all possible scenarios and discuss the generic cases here.

Small $k_2 \sin\alpha$. If ρ_3 and α_3 are of order 1, (12) shows that $(k_2/k_+)\sin\alpha$ is $\mathcal{O}(\epsilon^2)$ or smaller. For $(k_2/k_+)\sin\alpha = \mathcal{O}(\epsilon^2)$ the triplet phase θ must be of order 1. Smaller values of θ are correlated with even tinier values of α. Interestingly, one now finds the relation:

$$2\rho_3 - \rho_1 = \mathcal{O}(\epsilon^2). \quad (13)$$

That is, in this scenario the electroweak scale is defined by $k_+^2 = \mathcal{O}((2\rho_3 - \rho_1)v_R^2)$, while v_R^2 is as usual defined by μ_3^2/ρ_1 through (7). In any GUT scenario in which Δ_L and Δ_R belong to the same GUT multiplet, $2\rho_3 - \rho_1$ vanishes exactly above the GUT scale. Below the GUT scale $2\rho_3 - \rho_1$ acquires a small non-zero value from radiative corrections. Such a situation occurs for example in SO(10) models [10]. The responsibility of the GUT symmetry for the gauge hierarchy $k_+ \ll v_R$ is certainly very interesting. However, the small parameter $2\rho_3 - \rho_1$ also proliferates into the mass matrices, the leading terms for the singly charged masses are as in (10). While the doublet with the FCNC Higgs now becomes heavy, we instead face an extra light Higgs triplet, whose components are dominantly Δ_L fields. The explicit calculation of the neutral mass matrix, however, shows that the triplet masses are of order v_L or smaller. Since at least one of the triplet fields couples to the Z-boson, this scenario would have been discovered at LEP-I. These findings also hold for the case that $k_2 \sin\alpha$ is exactly zero with $\theta \neq 0$.

If both $(k_2/k_+)\sin\alpha$ and α_3 are $\mathcal{O}(\epsilon)$, we find three light neutral Higgses: one is SM-like one and two are mixtures of the δ_L^0 and the flavor-violating ϕ_+^0. This scenario interpolates between the previous and the following one.

Large $k_2 \sin\alpha$. For $(k_2/k_+)\sin\alpha = \mathcal{O}(1)$ the minimization conditions in (12) require $\alpha_3 = \mathcal{O}(\epsilon^2)$, thus α_3 defines the gauge hierarchy here. This scenario agrees qualitatively with the one discussed above for $v_L = 0$. As stated previously the smallness of α_3 leads to a 2HDM in the decoupling limit, with unacceptable flavor-violating neutral couplings in (5). We have also calculated the renormalization group flow of the flavor-changing neutral Higgs couplings in order to rule out a possible suppression mechanism of these couplings.

No SCPV. For $\alpha = \theta = 0$ the minimization conditions with respect to Δ_L^{0i} and ϕ_2^{0i}, which implied the smallness of α_3 or $2\rho_3 - \rho_1$, vanish and we are just left with 4 minimization conditions. Now we can arrange the non-standard Higgses to decouple for $v_R \to \infty$. Also this scenario agrees qualitatively with the zero phase case with $v_L = 0$: (7) implies a severe fine-tuning problem, after eliminating v_R and k_2/k_1 from (7) the parameters $\mu_{1,2,3}$, $\alpha_{1,2,3}$ and ρ_1 combine to a relation requiring that parameters of order 1 cancel up to terms of order ϵ^2. This phenomenon is at the heart of the gauge hierarchy problem. One usually addresses it by choosing the mass parameters $\mu_{1,2}$ of the fields which break the electroweak symmetry to be of order k_+. That is, the gauge hierarchy $k_+ \ll v_R$ is put into V by hand. In our case this solution would also require to choose $\alpha_i = \mathcal{O}(\epsilon^2)$, i=1,2,3, (see (7)) leading again to a 2HDM.

CP violation in V at a high scale. The essential prerequisite for the connection between the CP phases and the Higgs spectrum found above is the spontaneous breakdown of CP at the *electroweak* scale. Within the MLRM a new situation occurs, if one allows for explicit CP violation in V. In this case only the term involving α_2 in (2) changes (see Eq. (A.2) of [6]) and α_2 may be complex. To order ϵ^0 this case can be mapped on the discussed one by rescaling $\phi_2^0 \to \phi_2^0 e^{-i\varphi}$ with $\varphi = \arg(\alpha_2 v_R^2/2 - \mu_2^2)$. Hence the previous findings on the Higgs spectrum in the decoupling limit remain valid with the replacement $\alpha \to \alpha + \varphi$. Now the SM-like spectrum occurs for $\alpha = -\varphi$. Going beyond the MLRM, one can add extra singlet Higgs fields with large VEVs of order v_R [8]. Then spontaneous CP phases can appear between different VEVs of order v_R, i.e. CP is broken well above the electroweak scale. This case can be mapped on the case with explicit CP violation, with φ now being a calculable function of the new CP phases. Phenomenological studies of CKM CP violation in these models requires a renormalization group analysis of the Yukawa sector to account for the large logarithm $\ln(v_R/k_+)$.

Conclusions. We have determined the general Higgs potential of the MLRM in the decoupling limit $v_R \to \infty$ and established a strict connection between the CP properties of the vacuum state and the Higgs spectrum: if any of the CP phases $\alpha + \varphi$ or θ is nonzero, the low-energy model always differs from the SM. Either a 2HDM with flavor-changing Higgses or a model with additional light triplet fields emerges, both of which are phenomenologically ruled out. The appearance of the triplet Higgs model in the decoupling limit for small but non-zero α has not been discussed in previous analyses. In particular papers which simultaneously assume a large $\alpha + \varphi$ and a SM-like Higgs spectrum are not correct. An important difference to previous analyses of the Higgs sector is that our results hold generally, even for fine-tuned parameters. We found that fine-tuning arguments do not discriminate between scenarios with and without spontaneous CP phases. Fine-tuning is unavoidable for the Higgs potential to produce the gauge hierarchy $v_R \gg k_+$. To obtain the SM in the decoupling limit in the case $\alpha+\varphi = \theta = 0$, a fine-tuning condition between $\mu_{1,2,3}$, $\alpha_{1,2,3}$ and ρ_1 implied by (7) must be fulfilled. This condition requires parameters of order v_R^2 to cancel up to terms of order k_+^2 and cannot be justified by an approximate symmetry. The main conclusion of our paper is that the MLRM does not allow for spontaneous CP violation at the electroweak scale. Adding extra singlet fields does not change this conclusion, but opens the possibility for spontaneous CP violation at the high scale v_R parametrized by a new phase φ. Decoupling of the non-standard Higgses then requires $\alpha = -\varphi$.

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